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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 1, 2019/2020

### ETM7166 – DIGITAL SIGNAL PROCESSING SYSTEMS AND DESIGN IN TELECOMMUNICATIONS ( All sections / Groups )

24 OCTOBER 2019  
1:00 PM – 4:00 PM  
( 3 Hours )

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#### INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages with 4 Questions only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

### Question 1

- (a) Digital signal processing (DSP) has been used in a wide range of applications.
- Briefly describe two advantages of DSP compared to analogue signal processing. [4 marks]
  - Briefly describe the options available to implement DSP systems in practice. [4 marks]
  - Figure Q1.1 shows the spectrum of a bandlimited analogue signal. Illustrate the spectrum of a sampled version of this signal if ideal sampling is performed at a rate higher than the Nyquist rate. [4 marks]

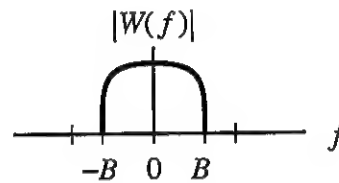


Figure Q1.1

- (b) Consider the system in Figure Q1.2

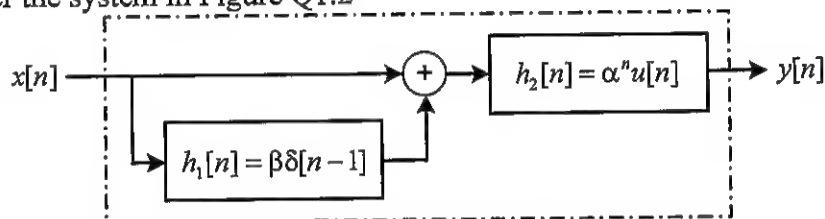


Figure Q1.2

- Find the impulse response  $h[n]$  of the overall system. [4 marks]
  - Is the system causal? Justify your answer. [2 marks]
- (c) Consider the following system function:

$$H(z) = \frac{5 - 1.8z^{-1}}{1 - 0.8z^{-1} + 0.12z^{-2}} = \frac{2}{(1 - 0.2z^{-1})} + \frac{3}{(1 - 0.6z^{-1})}$$

- Determine the linear constant coefficient difference equation (LCCDE). [2 marks]
- Determine the impulse response of the system  $h[n]$ , assuming the system is causal. [3 marks]
- Determine whether the filter is realizable. [2 marks]

Continued...

## Question 2

- (a) (i) Briefly discuss the differences between the discrete-time Fourier transform (DTFT) and the discrete Fourier transform (DFT). [4 marks]
- (ii) Figure Q2 shows a practical approach to performing spectral analysis on a continuous-time signal  $g(t)$  by computing the DTFT,  $G(e^{j\omega})$  of its discrete-time equivalent  $g[n]$ . The DTFT is usually approximated by the DFT operation. In this context, explain the difference between *high-density spectrum* and *high-resolution spectrum*. [4 marks]

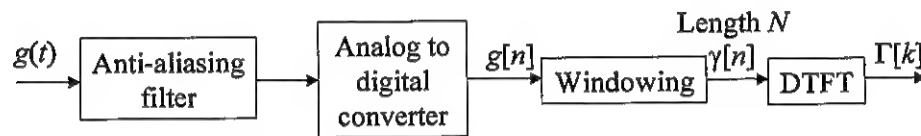


Figure Q2

- (b) In the context of designing digital finite impulse response (FIR) filters, answer the following questions.
- (i) Explain the rationale of truncating the impulse response of an ideal FIR digital filter. [2 marks]
- (ii) With the aid of a diagram, explain the effects caused by this truncation. Suggest a method to minimize them. [3 marks]
- (c) A signal is sampled at a frequency of 1 kHz, has a useful content from 0 to 100 Hz and is corrupted with noise from 400 to 500 Hz. Design a digital FIR low-pass filter based on a fixed window to attenuate the noise by at least 53 dB without affecting the useful content by more than 3 dB. [7 marks]
- (d) A low pass digital filter has the following characteristics (equiripple passband, but monotone stopband):

$$0.95 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq \omega \leq 0.3\pi$$

$$|H(e^{j\omega})| \leq 0.15, \quad 0.4\pi \leq \omega \leq \pi$$

By using the bilinear transformation method, determine the minimum order for the filter. [5 marks]

Continued...

### Question 3

- (a) Explain 3 reasons why we need to consider many different structures when designing discrete-time systems. [6 marks]
- (b) Figure Q3(a) shows a non-canonic implementation of an Infinite Impulse Response (IIR) filter. Suggest a simple modification in the structure in order to make the system canonic.

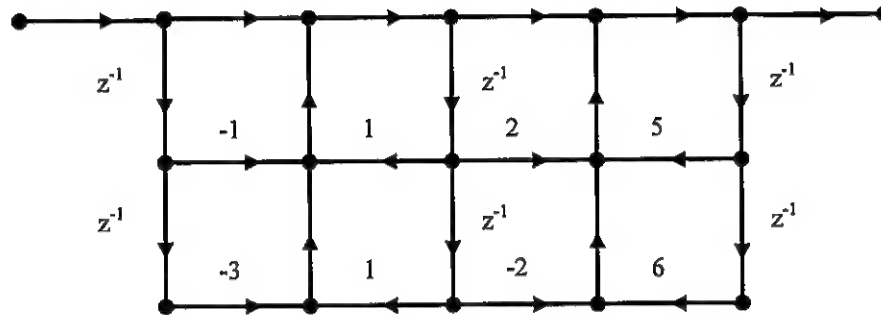


Fig. Q3(a)

[3 marks]

- (c) Consider the interpolator structure in Fig. Q3(b). The filter  $H(z)$  has a transfer function  $H(z) = 3z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 3z^{-5} + z^{-6} + 5z^{-7} + 5z^{-8}$

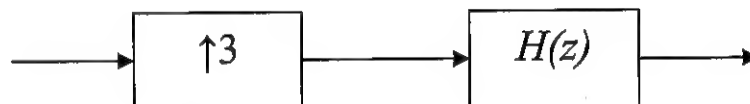


Fig. Q3(b)

- (i) The multirate system will be more efficient if the filter  $H(z)$  is represented in polyphase form. Derive the 3-branch polyphase decomposition of  $H(z)$  for the above system. Show explicitly the transfer function of each subfilter  $E_m(z)$ . [3 marks]
- (ii) Realize the system in Fig. Q3(b) using a 3-branch polyphase decomposition. Design your system so that the computational complexity will be lowest. [2 marks]
- (d) Digital audio tape (DAT) drives use a sampling frequency of 48kHz, whereas compact disc (CD) players operate at a rate of 44.1kHz. In order to record directly from a CD onto a DAT, it is necessary to convert the sampling rate from 44.1kHz to 48kHz.
- (i) Draw the block diagram of an efficient sampling rate converter for the system. [2 marks]

Continued...

- (ii) Determine the frequency response of the filter necessary to suppress imaging and ensure the absence of aliasing in the system. [2 marks]
- (iii) Suggest whether FIR or IIR filter is more computationally efficient for the above sampling rate conversion. Explain your answer. [3 marks]
- (e) A noisy speech signal, after passing through an unknown system, manages to have most of its noises removed. With the aid of a suitable diagram, suggest how adaptive filters can be used to identify the properties of the unknown system. [4 marks]

#### Question 4

- (a) Figure Q4(a) shows the block diagram of a Linear Prediction Coding (LPC) decoder. Explain how the bit stream is decoded into a synthetic speech.

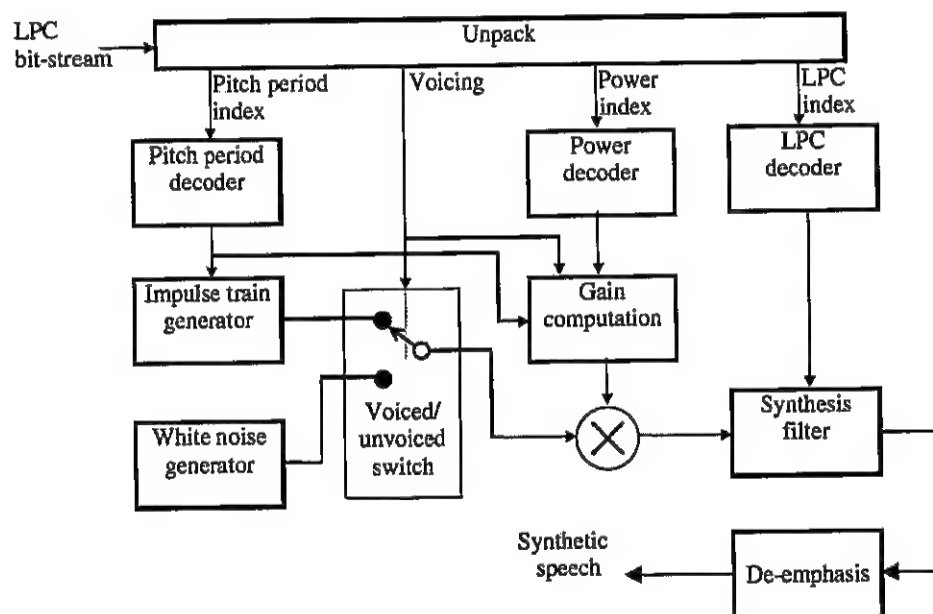


Figure Q4(a)

[8 marks]

- (b) Briefly describe the use of energy, zero crossing rates, and prediction gain in the voicing detector of an LPC model. [6 marks]
- (c) Figure Q4(b) shows the plot of a voiced segment with its corresponding autocorrelation function. Explain how the pitch period can be estimated from each plot. Estimate the pitch period. [3 marks]

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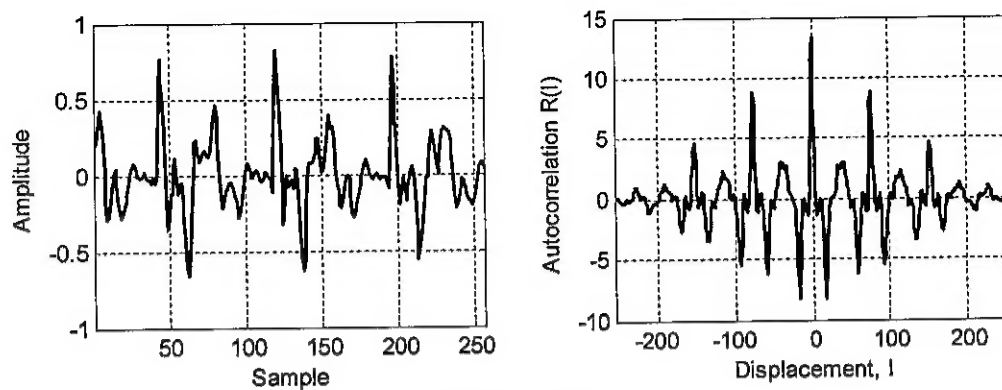


Figure Q4(b)

- (d) A 256-sample voiced segment, with 16 bits/sample, is to be coded using parametric coding. The linear prediction filter employed is of 14<sup>th</sup> order, with each coefficient represented by 4 bits, and a combined pitch period, power and voicing bit of 8 bits. Determine the compression ratio achieved by this parametric coder. [3 marks]
- (e) In linear prediction applications, it is required to estimate the current sample of the input sequence as a linear combination of the past  $M$  input samples. With the aid of a diagram, explain how adaptive filters can be used to achieve this. [5 marks]

**End of Questions**

## Appendix: Formula Sheet

### The $z$ -transform

Properties of the  $z$ -transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n] + by[n]$	$aX(z) + bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n - m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common  $z$ -transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n - n_0]$	$z^{-n_0}$	Possibly $\forall z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $

Closed-form Expression for Some useful Series

$\sum_{n=0}^{N-1} a^n = \frac{1 - a^N}{1 - a}$ $\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$ $\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$ $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad  a  < 1$	$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \quad  a  < 1$ $\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$ $\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$
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## FIR Filter Design

Ideal bandpass

$$h[n] = \frac{w_2}{\pi} \operatorname{sinc}\left(\frac{w_2(n - M/2)}{\pi}\right) - \frac{w_1}{\pi} \operatorname{sinc}\left(\frac{w_1(n - M/2)}{\pi}\right),$$

$$n = 0, 1, \dots, M$$

Fixed windows

Window	Window function
Rectangular	$w[n] = 1$
Hann	$w[n] = 0.5 - 0.5 \cos(2\pi n/M)$
Hamming	$w[n] = 0.54 - 0.46 \cos(2\pi n/M)$
Blackman	$w[n] = 0.42 - 0.5 \cos(2\pi n/M) + 0.08 \cos(4\pi n/M)$

Window	Passband ripple $20 \log_{10} \delta_p$	Stopband attenuation $20 \log_{10} \delta_s$	Transition width $ w_p - w_s $
Rectangular	-13	-21	$1.8\pi/M$
Hann	-31	-44	$6.2\pi/M$
Hamming	-41	-53	$6.6\pi/M$
Blackman	-57	-74	$11\pi/M$

Kaiser window

$$w[n] = \frac{I_0(\beta(1 - (n/\alpha - 1)^2)^{0.5})}{I_0(\beta)}$$

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50 \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21), & 21 \leq A \leq 50 \\ 0, & A < 21 \end{cases}$$

$$M = \begin{cases} (A - 7.95)/(2.285\Delta w), & A \geq 21 \\ 5.655/\Delta w, & A < 21 \end{cases}$$

Optimal Filter Design (Parks-McClellan Algorithm) Estimated Filter Order

$$N = \frac{-20 \log \sqrt{\delta_p \delta_s} - 13}{14.6 \Delta f}$$

Continued...



## CLASSIFICATION OF LINEAR-PHASE FIR SYSTEMS

	$h[n]$ symmetric: $h[n] = h[N-n]$	$h[n]$ antisymmetric: $h[n] = -h[N-n]$
$N$ even	<b>Type I Linear Phase Filter</b> $H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=0}^{N/2} a[k] \cos(k\omega)$ $a[0] = h[N/2]$ $a[k] = 2h[(N/2) - k]$	<b>Type III Linear Phase Filter</b> $H(e^{j\omega}) = je^{-jN\omega/2} \sum_{k=1}^{N/2} c[k] \sin(k\omega)$ $c[k] = 2h[(N/2) - k]$
$N$ odd	<b>Type II Linear Phase Filter</b> $H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} b[k] \cos((k-1/2)\omega)$ $b[k] = 2h\left[\frac{(N+1)}{2} - k\right]$	<b>Type IV Linear Phase Filter</b> $H(e^{j\omega}) = je^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} d[k] \sin((k-1/2)\omega)$ $d[k] = 2h\left[\frac{(N+1)}{2} - k\right]$

Continued...

## IIR Filter Design

Normalized Butterworth lowpass

$N$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$
1	1.000							
2	1.414	1.000						
3	2.000	2.000	1.000					
4	2.613	3.414	2.613	1.000				
5	3.236	5.236	5.236	3.236	1.000			
6	3.864	7.464	9.142	7.464	3.864	1.000		
7	4.494	10.10	14.59	14.59	10.10	4.494	1.000	
8	5.126	13.14	21.85	25.69	21.85	13.14	5.126	1.000

Filter order

$$d = \left( \frac{(1 - \delta_p)^{-2} - 1}{\delta_s^{-2} - 1} \right)^{0.5}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

Design	Filter order
Butterworth	$N \geq \frac{\log d}{\log k}$
Chebyshev I, II	$N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$
Elliptic	$N \geq \frac{\log(16/d^2)}{\log(2/u)}$
	$u = \frac{1 - (1 - k^2)^{0.25}}{1 + (1 - k^2)^{0.25}}$

$$\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$$

Continued...

## Frequency transformations

Target class	Transformation	Edge frequencies of target class
Highpass	$s \rightarrow \frac{\Omega_p \Omega'_p}{s}$	$\Omega'_p$
Bandpass	$s \rightarrow \Omega_p \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$	$\Omega_l, \Omega_u$
Bandstop	$s \rightarrow \Omega_p \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$	$\Omega_l, \Omega_u$

## Impulse invariance transformation

$$H_a(s) = \sum_{k=0}^{p-1} \frac{A_k}{s - s_k} \longrightarrow H(z) = \sum_{k=0}^{p-1} \frac{T_s A_k}{1 - e^{s_k T_s} z^{-1}}$$

## Bilinear transformation

$$H(z) = H_a(s) \Big|_{s = \frac{2}{T_s} \frac{1-z^{-1}}{1+z^{-1}}}$$

$$\Omega = 2 \tan(\omega/2)/T_s$$

Continued...

## Discrete-time Fourier Analysis

The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1 x_1[n] + A_2 x_2[n]$	$A_1 X_1[k] + A_2 X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k] W_N^{kn_0}$
Frequency shifting	$x[n] W_N^{-kn_0}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \otimes y[n]$	$X[k] Y[k]$
Modulation	$N x[n] y[n]$	$X[k] \otimes Y[k]$

**End of Paper**

